Int. J. Heat Mass Transfer. Vol. 3, pp. 133–154. Pergamon Press 1961. Printed in Great Britain.

INFLUENCE OF TURBULENCE ON THE TRANSFER OF HEAT FROM PLATES WITH AND WITHOUT A PRESSURE GRADIENT

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(*Received* 1 *February* 1961)

Abstract—The work described in this paper is a continuation of that done by Kestin and Maeder on the influence of free-stream turbulence on the coefficient of heat transfer from cylinders in cross-flow. The present work is concerned exclusively with the flat plate and deals at length with the case of zero incidence, i.e. of zero pressure gradient. The effect of adding a favorable pressure gradient was investigated in a preliminary way.

Kestin and Maeder have demonstrated the existence of two effects produced by an increase in the turbulence intensity of the free-stream in the case of cross-flow past a circular cylinder. (1) An increase in turbulence intensity causes earlier transition, and, generally, affects the flow pattern about the body. (2) An increase in turbulence intensity causes *local* changes in the coefficients of heat transfer and, presumably, in the flow pattern in the boundary layer. The existence of the local effect has also been observed by Kestin, Maeder and Sogin, Giedt, Sato and Sage, Sage *et al.*, Seban, and van der Hegge Zijnen.

The present investigation shows that the local effect is completely absent in the case of a flat plate at zero incidence, in good agreement with the work due to Edwards and Furber and Kline *et al.*, but in contradiction to the findings of Sugawara and Sato. This is a remarkable difference between the present case and that of a cylinder. A qualitative explanation of this difference was suggested by Kestin *et al.* The main conclusion reached by them was that large effects from changes in the free-stream turbulence can be produced only in the presence of pressure gradients. This conclusion was tested by imposing a favorable pressure gradient on the plate. It was found that small changes in the turbulence intensity of the free stream cause large changes in the coefficient of heat transfer in the laminar range. Experiments with a pressure gradient were carried out in a preliminary way, and no exhaustive measurements were undertaken at this stage. In particular, the effect on turbulent boundary layers as well as the effect of adverse pressure gradients were not investigated.

The experimental arrangement and the techniques used in measurement are described in detail; the accuracy is carefully examined and detailed check-measurements, in particular, measurements of velocity profiles were undertaken in order carefully to correlate the heat transfer measurements with the different flow regimes which are possible in the boundary layer.

Résumé—Le travail décrit dans cet article fait suite à celui de Kestin et Maeder sur l'influence de la turbulence de l'écoulement libre sur le coefficient de transmission de chaleur de cylindres placés normalement à l'écoulement. Le présent travail concerne exclusivement le cas de la plaque plane et traite en détail le cas de l'incidence nulle, c'est-à-dire du gradient de pression nul. L'effet de l'addition d'un gradient de pression favorable a été étudié de façon préliminaire.

Kestin et Maeder ont démontré, dans le cas de l'écoulement perpendiculaire à un cylindre, que l'augmentation de l'intensité de la turbulence dans l'écoulement libre avait pour effet:

1° d'avancer la transition et de modifier l'écoulement autour du corps;

2° d'affecter localement les coefficients de transmission de chaleur et probablement aussi l'écoulement dans la couche limite. L'existence d'un effet local a également été observé par Kestin, Maeder, Sogin, Giedt, Sato et Sage, Sage et ses collaborateurs, Seban et van der Hegge Zijnen.

La présente recherche montre que l'effet local est inexistant dans le cas d'une plaque plane à incidence nulle. Cette conclusion est en bon accord avec le travail de Edwards, Furber, Kline et ses collaborateurs, mais en contradiction avec les résultats de Sugawara et Sato. C'est une différence

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remarquable entre le cas présent et celui du cylindre. Une explication qualitative de cette divergence a été suggérée par Kestin, Maeder et Wang. Leur conclusion principale est que la variation de la turbulence du courant libre n'a d'influence notable que s'il existe des gradients de pression. Cette conclusion a été contrôlée dans le cas de la plaque, en imposant un gradient de pression favorable. On a trouvé que des petites variations de l'intensité de la turbulence entraînaient de grandes variations du coefficient de transmission de chaleur dans la zone laminaire. Les expériences effectuées avec gradient de pression sont préliminaires, aucunes mesures complètes n'ont encore été entreprises à ce stade. En particulier, l'effet sur les couches limites turbulentes aussi bien que l'influence de gradients de pression défavorables n'ont encore été étudiés.

L'appareillage et les techniques utilisés pour les mesures sont décrits en détail; la précision est étudiée avec soin et les mesures de contrôle bien détaillées, en particulier, les mesures des profils de vitesse ont été effectuées pour les comparer aux mesures de transmission de chaleur pour les différents régimes d'écoulement dans la couche limite.

Zusammenfassung—Die Arbeit von Kestin und Maeder über den Einfluss der Freistromturbulenz auf den Wärmeübergangskoeffizienten bei Zylindern im Querstrom wird hier fortgesetzt für die ebene Platte im Fall der Null-Inzidenz d.h. für einen Druckgradienten von Null. In vorläufigen Versuchen wird der Einfluss günstiger Druckgradienten untersucht. Beim querangeströmten Kreiszylinder haben Kestin und Maeder zwei Effekte nachgewiesen, die von einer Erhöhung der Turbulenzintensität des Anblasstromes stammen. (1) Die Erhöhung der Turbulenzintensität verursacht einen vorzeitigeren Umschlag und beeinflusst ganz allgemein das Strömungsbild in der Umgebung des Körpers. (2) Die Erhöhung der Turbulenzintensität bedingt örtliche Veränderungen des Wärmeübergangskoeffizienten und wahrscheinlich auch des Strömungsbildes in der Grenzschicht. Diesen lokalen Effekt stellen auch Kestin, Maeder und Sogin, Giedt, Sato und Sage, Sage und andere, Seban und van der Hegge Zijnen fest.

Die vorliegende Untersuchung zeigt, dass ein lokaler Effekt bei der ebenen Platte vollkommen fehlt, was mit der Arbeit von Edwards und Furber und Kline und andere übereinstimmt, jedoch den Ermittlungen von Sugawara und Sato widerspricht. Dies bedeutet eine bemerkenswerte Abweichung des hier behandelten Falles vom Zylinder. Eine qualtitative Erklärung der Verschiedenheit geben Kestin, Maeder und Wang. Ihre Schlussfolgerung besagt, dass grosse Effekte durch Änderung der Freistromturbulenz nur in Gegenwart von Druckgradienten hervorgerufen werden können. Dies wurde mit Hilfe eines günstig gewählten Drucksgradienten an der Platte geprüft. Es ergab sich, dass kleine Änderungen der Turbulenzintensität des Freistromes grosse Änderungen des Wärmeübergangskoeffizienten im laminaren Bereich hervorrufen. Die Versuche mit dem Druckgradienten waren nur vorläufiger Art, es wurden noch keine eingehenden Messungen durchgeführt. Insbesondere ist noch nicht der Einfluss auf turbulente Grenzschichten und der Effekt weniger günstiger Druckgradienten untersucht.

Die Versuchseinrichtung und die Messmethode werden im Einzelnen beschrieben. Eingehende Kontrollmessungen sorgten für gute Genauigkeit. Besonders exakt wurden die Geschwindigkeitsprofile ermittelt, um die Wärmeübergangsmessungen mit den verschiedenen, in der Grenzschicht möglichen Strömungsarten in Beziehung setzen zu können.

Аннотация—Материалы, изложенные в настоящей статье, являются продолжением работы, проделанной Кестиным и Мидером по вопросу о влиянии турбулентности набегающего потока на коэффициент теплоотдачи цилиндров. В данной работе приведены результаты исследования теплоотдачи плоской пластины при условии, когда угол атаки равен нулю. В этом случае градиент давления равен нулю. Влияние величины градиента давления исследовалось отдельно.

Кестин и Мидер показали существование двух эффектов, которые возникают в результате повышения интенсивности турбулентности набегающего потока для случая, когда цилиндр расположен поперёк. (1) Повышение интенсивности турбулентности вызывает более ранний переход и, вообще, влияет на структуру потока над телом (2) Повышение интенсивности турбулентности вызывает локальные изменения в коэффициентах теплообмена, повидимому, в структуре потока в пограничном слое. Это также наблюдали Кестин, Мидер и Зогин, Гидт, Сато и Саге, Саге и другие, Себан и Ван дер Хегге Цинен.

Настоящее исследование показывает хорошее совпадение с данными, приведенными в работах Эдвардса, Фурбера, Клайна и др., но противоречит данным, опубликованным в работах Сагавара и Сато, для случая плоской пластины с углом атаки равным нулю при полном отсутствии локального эффекта. В этом заключается существенное различие между настоящим случаем и случаем с цилиндром. Качественное объяснение этого различия было предложено Кестиным, Мидером и Уэнгом. Основной вывод, к которому они пришли, заключается в следующем: большие эффекты, возникающие при изменении турбулентности набегающего потока, могут иметь место только при наличии градиентов давления. Этот вывод проверялся путём введения подходящего градиента давления. Было найдено, что небольшие изменения в интенсивности турбулентности набегающего потока вызывают большие изменения в коэффициенте теплообмена в ламинарной области. Опыты с градиентом давления проводились приближённым методом, а тщательные измерения не проводились на этом этапе. В частности, влияние на турбулентные пограничные слои, а также влияние неблагоприятных градиентов давления, не исследовалось.

В статье дано детальное описание экспериментальной установки. Точность её проверена, и, в частности, были проведены особо точные измерения профилей скорости для корреляции теплообмена с различными режимами, которые могут иметь место в пограничном слое.

LIST OF SYMBOLS

A, area of heater;
b, half-width of heater;
C, numerical factor in equation (31);
C_1, numerical factor in equation (10);
C_2, numerical factor in equation (12);
C_b, Stefan–Boltzmann radiation constant;
C_f, coefficient of skin friction;
f, correction factor, equation (27);
I, current flowing through heater;
k, thermal conductivity (average);
k_T, thermal conductivity (at temperature T);
L, length of plate;
Nu, Nusselt number (based on length co-ordinate for plate);
P_{atm}, atmospheric pressure;
P_r, radiation correction;
P_s, static pressure;
$P_{s\infty}$, static pressure in free stream;
P_t, total pressure;
q_∞, dynamic pressure in free stream;
Re, Reynolds numbers (based on length co-ordinate for plate);
r, pressure ratio, equation (3);
T, temperature;
T_a, air temperature;
$T_{a\infty}$, air temperature in free stream;
T_w, wall temperature;
ΔT, temperature difference ($=T_w - T_{a\infty}$);
Tu, intensity of turbulence (based on longitudinal oscillating component);
U_∞, free stream velocity;
u, longitudinal velocity component in boundary layer;
u', longitudinal fluctuating velocity component;
V, voltage across heater;
x, co-ordinate along plate in flow direction;
y, co-ordinate normal to plate.

Greek symbols

α, coefficient of heat transfer (mean value over heater);
α_1, coefficient of heat transfer (at center-line of heater);
Δ, dimensionless pressure gradient, equation (4);
δ, boundary layer thickness (calculated);
δ_m, boundary layer thickness (measured at $u/U_\infty = 0{\cdot}99$);
ϵ, emissivity;
ν, kinematic viscosity (average);
ν_T, kinematic viscosity (at temperature T);
ρ_∞, density in free stream.

1. INTRODUCTION AND RELATED WORK

IT HAS been known for some time that many experiments on forced convection show considerable, and inexplicable discrepancies in their results. It was suspected that some unknown factors must have been left out of account, in particular, the effect of the turbulence intensity of the free stream.

The present investigation is a continuation of that due to Kestin and Maeder [1] who gave an extensive discussion on the subject and reported experimental results on the overall coefficient of heat transfer from an "infinite" circular cylinder in cross-flow. The experiments showed unexpectedly large increases in the mean Nusselt number; for example, at a mean Reynolds number of 180 000, an increase in the mean Nusselt number of 14·3 per cent was observed for a change of turbulence intensity from 0·75 to 2·60 per cent only.

It is clear that the mean Nusselt number must be affected indirectly by the shifts of the points of transition and separation caused by different turbulence intensities in the free stream. However, by the use of tripping wires, these authors demonstrated conclusively that, in addition, there exists a local effect. In other words, having fixed the flow pattern in the boundary layer about a cylinder by the use of tripping wires, and having made it insensitive to changes in turbulence intensity, these authors found that, nevertheless, small changes in turbulence intensity still caused large changes in the Nusselt number. For example, again at a mean Reynolds number of 180 000, the mean Nusselt number increased by 26 per cent when the intensity of turbulence was varied from 0·75 to 2·66 per cent.

This local effect was subsequently measured directly by Kestin *et al.* [2]. It was also noted in the earlier work by Giedt [3, 4], Sato and Sage [5] as well as in the subsequent work due to Sage *et al.* [6, 7], Seban [8] and van der Hegge Zijnen [9].

The case of a flat plate was studied by Edwards and Furber [10] who have undertaken experiments to determine the influence of the free-stream turbulence on the *mean* coefficient of heat transfer from a flat plate provided with a round nose of very small radius. Two screens were used to promote turbulence intensities of 1·5 and 2·5 per cent, but these values were estimated rather than measured directly. The investigation led to the conclusion that changes in the intensity of turbulence of the external stream had no influence on the rate of heat transfer across a laminar or a turbulent boundary layer except insofar that a higher intensity promoted transition at a lower Reynolds number. This showed the existence of an important difference between the flat plate at zero incidence and cylinders or spheres. The present investigation was conducted to confirm this result and to elucidate the reasons for such a fundamental divergence of behavior.

At an earlier time, Fage and Falkner [11] had performed experiments on a thin, electrically heated platinum foil and found no influence of the free-stream turbulence on the mean coefficient of heat transfer in the laminar regime to which their measurements were confined.

The local coefficient of heat transfer from a flat plate to a turbulent air stream was measured by Sugawara and Sato [12] who employed a non-steady method. Their measurements showed no effect in the laminar region for intensities up to 1 per cent, in agreement with Edwards and Furber. However, in the turbulent range they measured large increases in the Nusselt number. In contradiction to Edwards and Furber, they measured increases of up to 55 per cent in the Nusselt number for a variation in turbulence intensity from 1·0 to 8·0 per cent. We shall revert to these measurements in section 7.

Finally, Reynolds *et al.* [13] made measurements on a flat plate in the presence of a turbulent boundary layer. The measurements were made in a tunnel of relatively high intensity of turbulence (1·5 to 5·0 per cent) and it was found that irrespectively of the level of turbulence, the Nusselt numbers satisfied von Kármán's equation (see equation (36) later) with an accuracy of ±4·5 per cent, i.e. with an accuracy of the order of their experimental error. These results can be interpreted as a confirmation of the findings due to Edwards and Furber. In addition to measuring rates of heat transfer, Reynolds *et al.* verified that the coefficient of skin friction was independent of the intensity of turbulence and agreed well with Schultz–Grunow's formula

$$\tfrac{1}{2}C_f = 1\cdot60\,(\ln Re)^{-2\cdot58}. \tag{1}$$

The relation of the latter measurement to the former will be appreciated if it is remembered that in incompressible flow, the velocity field is independent of the temperature field, but the temperature field is solely determined by the velocity field and the boundary conditions. Consequently, any changes in the temperature profiles must be preceded by changes in the velocity profiles, although the two effects may be of different orders of magnitude.

2. STATEMENT OF PROBLEM

From what has been said in the preceding section, it is clear that the investigation of the following two problems would be both important and interesting. *First*, enough doubt existed about the effect of turbulence intensity on the rate of heat transfer from a flat plate at zero incidence to warrant new measurements. It was also clear that *local* measurements would be the most conclusive.

Secondly, as it turned out, no effect other than a change in the transition Reynolds number was found, in full agreement with Edwards and Furber, and it became important to investigate the reasons for the fundamental difference in the behavior of flows past flat plates and those past cylinders or spheres. The difference in the flows passing over a cylinder and flat plate is the magnitude of the pressure gradient in the flow direction. At the forward portion of a cylinder or sphere there exists a strong favorable pressure gradient which is absent on a flat plate at zero incidence. Consequently, measurements were made on a flat plate with a positive pressure gradient imposed on it. Provisionally, and for the present, these measurements were confined to the laminar range only. This limitation was unavoidable, because a positive pressure gradient stabilizes the flow, and transition to a turbulent boundary layer could not be obtained with the present combination of tunnel speed and plate length.

A qualitative explanation of the part played by pressure gradients in the presence of free-stream oscillations was given by the present authors elsewhere [14]. It was shown that if free-stream oscillations are to produce an effective change in the boundary layer, the amplitude of the oscillation must be large, and must vary strongly in the main flow direction. As is known from numerous experiments, for example from the recent work by Crocker *et al.* [15], the amplitude of the oscillations in the free-stream is strongly affected by pressure gradients, particularly near a stagnation point.

In anticipation, it can be stated here that measurements on a flat plate with a pressure gradient imposed on it confirmed this hypothesis.

3. EXPERIMENTAL ARRANGEMENT

The measurements were carried out in the 22 in × 32 in Brown University low-speed tunnel described elsewhere [1]. The flat plate was placed vertically in the center-line of the tunnel. The local rate of heat transfer was measured electrically with the aid of two unit heaters at two positions along the plate. The combination of two heaters and variable wind velocity made it possible to cover a range of length Reynolds number from $Re = 35\,000$ to $Re = 600\,000$.

The unit heaters, exposing areas of 10 in × $\frac{1}{2}$ in to the air stream, were attached to the plate but insulated from it and heated electrically, the input into them being carefully metered. The flat plate was heated by steam which jacketed the unit heaters. The surface of the plate and the exposed surface of the heaters were maintained at a constant temperature thus eliminating all transfer of heat sideways. All temperatures were measured by copper–constantan thermocouples.

The turbulence intensity was increased by inserting a screen in the tunnel test-section upstream from the leading edge of the flat plate. The turbulence intensity was measured by a hot-wire anemometer and varied from 0·7 to 3·8 per cent. The scale of turbulence was not measured in the present experiment.

The pressure gradient along the flat plate and the velocity profiles of the boundary layers were measured with standard probes.

(a) *The flat plate*

A schematic diagram of the plate is shown in Fig. 1. The flat plate, 2 in thick, $20\frac{11}{16}$ in wide and $23\frac{25}{32}$ in high, consisted of a brass flat surface P, $\frac{1}{4}$ in thick and a brass cover B, $\frac{1}{16}$ in thick. The nose of the leading edge was round and had a radius of $\frac{1}{16}$ in. The brass cover joined the flat surface at $\frac{1}{2}$ in from the leading edge and right at the trailing edge through curved portions. Two end-pieces E, one on the top and one on the bottom, with taps of $1\frac{5}{32}$ in diameter were provided for circulating saturated steam to maintain a constant surface temperature on the plate. The jacket steam was obtained from a small laboratory boiler which was also described in Ref. 1. On the flat surface, two slots, each of 10·1 in × 0·6 in, were provided to accommodate the unit heaters a and b. The slots were about 6 in from the top and bottom walls of the tunnel test-section, thus eliminating any influence of the walls on the measurement of heat transfer. The first slot was $7\frac{3}{32}$ in from the leading edge and the second, $13\frac{19}{32}$ in from it. The design of the plate is shown in somewhat greater detail in Fig. 2.

The plate, owing to its 2 in thickness and the curved portions, aerodynamically formed an unsymmetrical wing section; the flow pattern over it was different from that over a "flat"

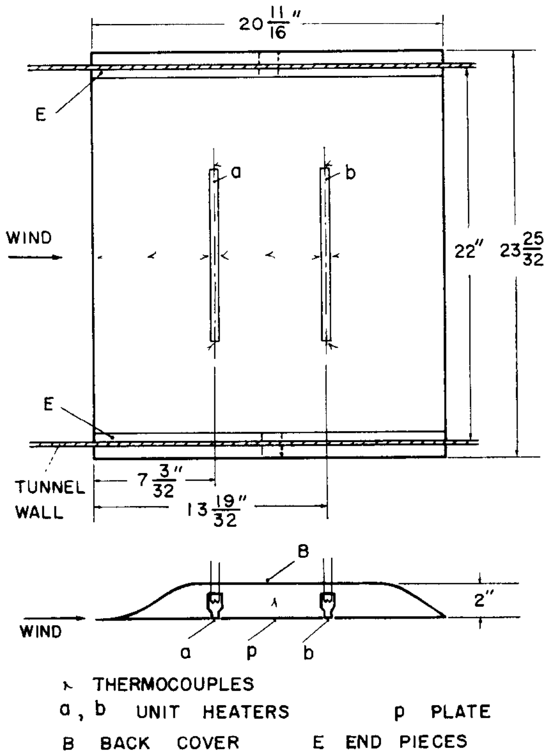


FIG. 1. Schematic drawing of flat plate.

plate at zero incidence and thickness. At the initial stage of the experiments, it was found that the flow separated at the leading edge and the pressure gradient along the plate was not negligible. In order to reproduce the flow pattern of a flat plate at zero incidence more precisely, two dividing plates P and B were provided, one upstream and one downstream, Fig. 3. The boundary layer formed on the front dividing plate was sucked away to the back of the plate through the suction slot. The pressure difference required for the suction was provided by four control louvers L mounted at the exit of the tunnel test-section. The control louvers were made of thin aluminium plates assembled in the form of a venetian blind and placed at approximately 40° to the direction of the flow as shown in Fig. 3. The front dividing plate was made of aluminum, $\frac{3}{8}$ in thick and 12 in wide and had a round nose and tapered trailing edge. The back dividing plate was made of wood, $\frac{3}{8}$ in $\times$ 9 in. With such an arrangement, a flat-plate flow pattern was successfully produced.

(b) *The unit heater*

The unit heater is first shown schematically in Fig. 4 and then in detail in Fig. 5. The heater was essentially a miniature boiler filled with saturated water and steam. It was made of copper and the heat-transferring surface e was 10 in $\times$ $\frac{1}{2}$ in. The heating element h was of the immersion type and had a rating of 30 W. During experiments the heating element was always kept immersed in water to prevent overheating. A baffle b was provided to cause circulation of the saturated water and steam inside the heater so that the surface temperature could be maintained uniform. The unit was charged with water from the bottom and the amount of water inside the heater could be examined by a gage m outside. Both the water inlet and the steam vent were provided with stop valves S which served to isolate the system during the experiments. The temperature of the steam inside the heater was measured by a thermocouple and the rate of heating was adjusted to provide a temperature equal to that of the jacket steam in the plate. Owing to the equal steam temperatures and to the provision of an insulating cover in the back of the heater, heat exchange between the heater and the jacket steam in the plate was reduced to

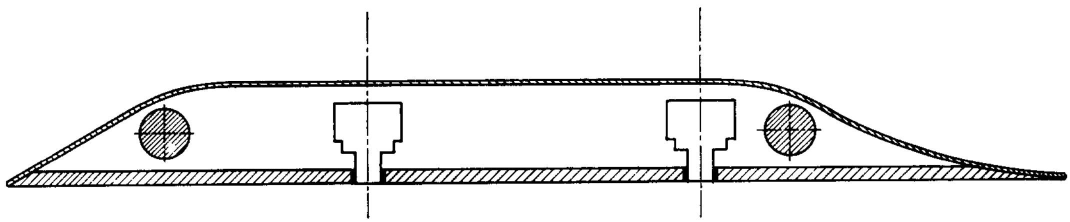


FIG. 2. Design drawing of flat plate.

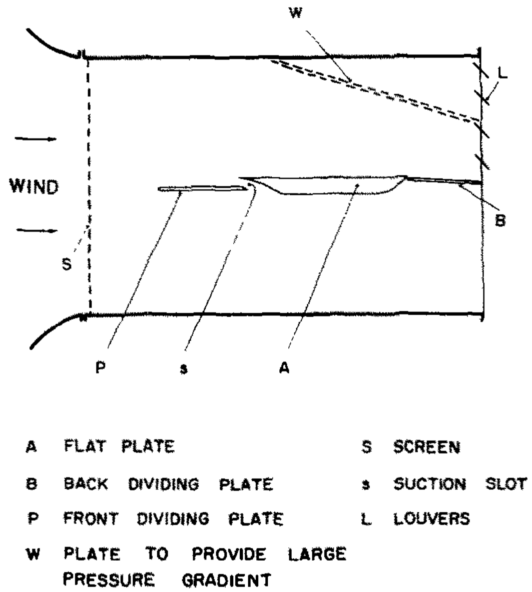


FIG. 3. Arrangement of plate in tunnel.

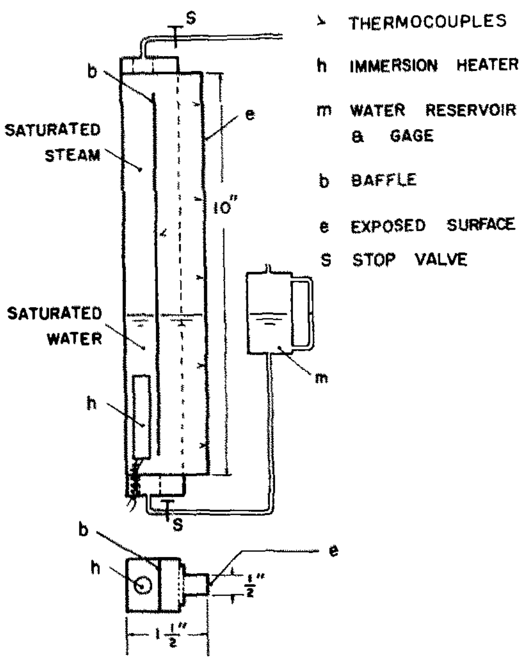


FIG. 4. Schematic drawing of unit heater.

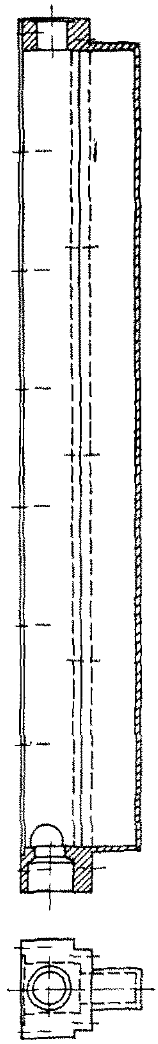
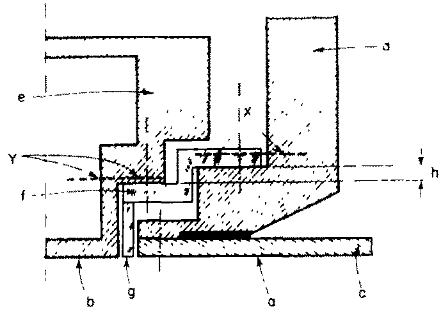


FIG. 5. Design drawing of unit heater.

a minimum. Therefore practically all of the electrical energy input into the heater was transferred through the exposed surface.

The unit heater was attached to the plate in such a way that its exposed surface was flush with that of the main plate at all temperatures. This was achieved by providing compensation for the different coefficients of linear thermal expansion of copper (heater) and brass (plate). The method of compensation can be best explained with reference to Fig. 6. It is seen from Fig. 6 that the unit heater e is attached to



c MAIN PLATE
d HEATER COVER
e UNIT HEATER
f TEFLON BRIDGE

FIG. 6. Method of compensation for thermal expansion.

its cover d through a Teflon bridge f; and the cover is screwed to the plate c. When the arrangement was assembled at room temperature, the exposed surface b of the heater was made flush with the plate surface a and the contact surface X was higher than the contact surface Y by an amount h. At higher temperatures all parts would expand. Taking surfaces a and b as reference, surfaces X and Y would move backwards and reach a position shown by the dotted lines, say. If without the Teflon bridge, the heater would be a separate unit and surface X would recede more than surface Y would, because the main plate would expand more due to the fact that the coefficient of thermal expansion of brass (plate) is higher than that of copper (heater). The difference was compensated by the expansion of the Teflon bridge through the length h. The value of the length h was estimated and found to be fairly small.

The gap g between the heater and the plate was also filled with Teflon. Being an insulating material, the Teflon insert cut down the amount of heat conduction sideways, thus eliminating the complication of correcting for it when a mean area was taken for the evaluation of the Nusselt number.

It must be remarked here that the excellent experimental results obtained, as will be seen later, were due in great measure to the successful design of the heater.

(c) *Power-input measurement*

The electrical power-input to the heaters was supplied from a 230-V a.c.–d.c. converter set. The undesirably large voltage fluctuations were screened out by a voltage stabilizer which was described in detail in Ref. 1. The d.c. output from the stabilizer was constant to within 0·1 per cent.

The measuring circuit was also the same as in the Ref. 1, where it was shown that the rate of heat transfer could be measured with an accuracy of 0·3 per cent.

(d) *Temperature measurement*

All temperatures were measured by 0·005 in diameter copper and constantan thermocouples. Before setting-up, one piece of the thermocouple wire was calibrated with the aid of a high-precision potentiometer and with reference to two precision, etched-stem, mercury-in-glass thermometers provided with Bureau of Standards certificates. The accuracy of the calibration was to 0·005°C in the room-temperature range and to 0·01°C near the steam point.

The temperature of the plate surface was measured at eleven positions and that of each heater surface at five positions as shown in Figs. 1 and 4 respectively. The thermocouples used to measure the temperature of the plate surface were attached to the plate in the following way (see also Ref. 1). The bared ends of the copper and constantan wires were soldered into two very fine holes drilled in the plate. The two holes were very close to each other so that the two wires could be considered being soldered together. The thermocouples to measure the temperature of the heater surface were attached to the heater in the same way except that only the constantan wires were soldered to it; the heater, being made of copper, served as the other wire.

Thermocouples to measure the jacket steam temperature and the temperature of the steam in each heater were also provided. The air temperature was deduced from the measurement of the stagnation temperature in the settling chamber of the wind tunnel.

All temperatures, except the air temperature, were measured relative to the jacket steam in the plate and the e.m.f. of the thermocouples was registered by a potentiometer recorder using a scale of 1 mV to 25 cm; in other words, differential measurements were made. Since all these temperatures were only slightly different from the jacket steam temperature a differential measurement was found very convenient and accurate.

The temperature of the jacket steam in the plate and the stagnation air temperature were measured independently by a portable potentiometer.

Owing to the successful design, the heaters had a surface temperature constant to within $\pm 0{\cdot}2$°C in most runs. In one or two runs at high speeds it rose to $\pm 0{\cdot}5$°C. The plate surface temperature was maintained constant to within $\pm 0{\cdot}5$°C at all positions except near the leading edge. Owing to the wedge-like shape of the leading edge where the heat transfer is extremely high, the supply of the jacket steam in the plate was insufficient to compensate for the cooling effect of the air-stream. Consequently the plate temperature near the leading edge varied from 92·6°C at the lowest speed to 67·3°C at the highest speed while the rest of the plate was always at nearly 100°C. The variation of the leading-edge temperature with the air speed is shown in Fig. 7. This was unavoidable and not very serious because the second thermocouple from the leading edge, $3\frac{1}{2}$ in away from it, already registered the normal plate temperature in all test runs. In view of the design of the leading edge, it was reasonable to assume a constant temperature, the value measured by the first thermocouple, $\frac{3}{4}$ in away from the leading edge, over a length of $1\frac{1}{2}$ in and then the normal plate temperature for the rest of the plate. The sudden jump, i.e. the discontinuity in temperature was assumed to occur at $x = 1{\cdot}5$ in. The rate of heat transfer was then corrected to the isothermal condition in the calculations of the Nusselt number according to the methods due to Eckert and Drake [16] and Rubesin [17]. The correction, as given in section 6, was only of the order of 5 per cent.

The heater surface temperature also differed somewhat from the plate surface temperature. In this case, however, the difference was only 1°C in most runs and rose to 3°C in a few runs at high speeds. Corrections were also applied by

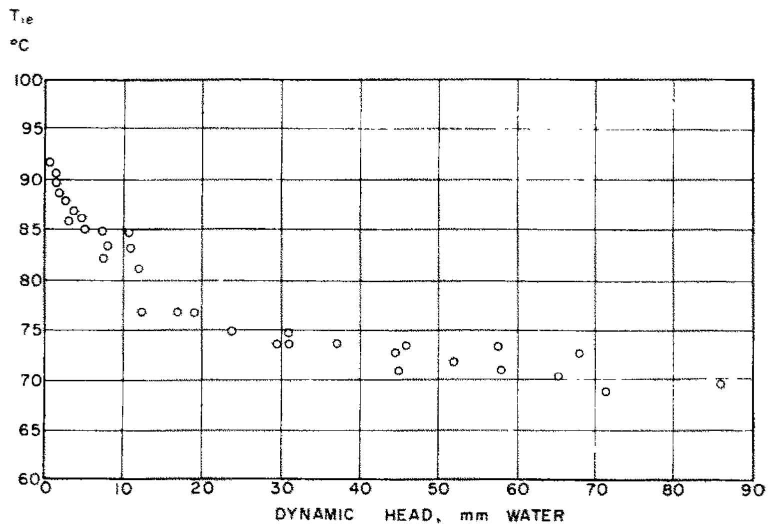


FIG. 7. Variation of temperature near leading edge.

the use of the same methods; the corrections turned out to be less than 1·5 per cent in all runs.

(e) *Measurement of turbulence intensity*

The turbulence intensity,

$$Tu = (\overline{u'^2})^{1/2}/U_\infty \qquad (2)$$

was measured by a hot-wire anemometer. All measurements were made at exactly the same flow conditions as in the heat transfer measurements but only after the latter had all been finished. It was done in such an order solely on the ground of convenience in practical operation.

In each measurement the probe (wire 0·00015 in in diameter) was placed opposite the heater at a distance of approximately 1·5 in from the surface of the plate, i.e. outside the boundary layer (the boundary layer thickness being of the order of 0·25 in). The turbulence intensity in the present wind tunnel remained approximately uniform for a given air speed throughout the entire test-section when it was clear. The uniformity was checked when the whole set-up was present in the test-section. Thus all the measurements were made in the presence of the flat plate and the dividing plates.

It was mentioned before that higher turbulence intensities were produced by placing a screen upstream of the plate. The screen S was of $\frac{3}{4}$ in mesh, 0·148 in diameter and placed 18·5 in upstream from the leading edge of the plate, as shown in Fig. 3. It is well known, e.g. as shown by Dryden *et al.* [18], that the turbulence intensity decreases in the downstream direction behind such a screen. They also found a perfect correlation between the local turbulence intensity and the screen-mesh length, Z/M, where Z is the distance downstream from the screen and M is the mesh size. Thus the value of the turbulence intensity at any position would be sufficient to determine the whole turbulence field. It is in the light of this fact that in the present experiments, when the turbulence screen was used, the turbulence intensity was also measured at one position only, i.e. opposite each heater and 1·5 in away from the flat plate. The measurements were also made when the flat plate and the dividing plates were present in the tunnel test-section.

As illustrated in Fig. 5 of Ref. 1, there were three fine screens in the settling chamber of the wind tunnel to equalize turbulence fluctuations. In the present experiments, one more fine screen was added at the entrance of the tunnel to reduce the amount of dust in the air before entering it. Moreover, all these screens were vacuum-cleaned at frequent intervals in order to gain a better control of the turbulence intensity and in order to prevent the hot-wire probes from becoming contaminated.

The accuracy of the turbulence measurements was believed to be well within 5 per cent.

4. AUXILIARY MEASUREMENTS

In addition to the measurements of the rate of heat transfer and temperature difference, it was necessary to perform measurements of the velocity profile in the boundary layer as well as to ascertain the deviation of the actual flow from the ideal flow as regards to the pressure gradient.

The static pressure in the free stream was investigated first. A standard static-pressure tube was employed. The static pressure in the transverse direction, i.e. normal to the flat plate, was found constant to within 0·5 per cent for all speeds with and without the turbulence screen. In the longitudinal direction, i.e. in the main stream direction, the static pressure varied slightly. The variation is shown in Fig. 8 in which the pressure ratio,

$$r = (P_s - P_{\mathrm{atm}})/(P_t - P_{\mathrm{atm}}) \qquad (3)$$

is drawn in terms of x, the distance from the leading edge. Here P_s P_t and P_{atm} denote the total, static and atmospheric pressures respectively. It is seen from Fig. 8 that the static pressure is highest near the leading edge. However, the maximum difference in the ratio r is only 0·025 over the entire length of the plate, which is considered negligible.

The static pressure on the surface of the plate was measured with the aid of three pressure taps, one near the leading edge, one between the two heaters and one near the trailing edge. The values of the ratio r for these measurements can also be seen in Fig. 8. They show a similar trend as that in the main stream, except near the

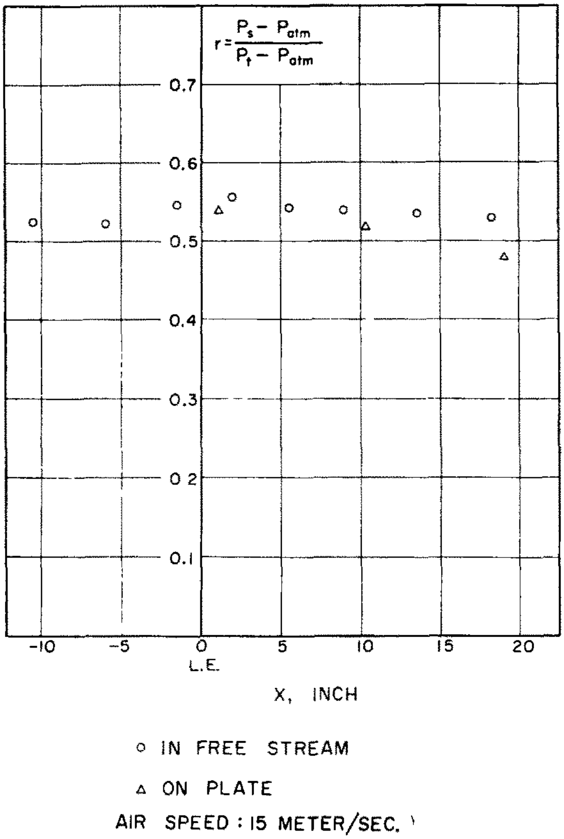


FIG. 8. Variation of static pressure; case of negligible pressure gradient.

trailing edge. The difference in the ratio r over the entire length of the plate is 0·06.

The pressure gradient can best be expressed by the following dimensionless quantity:

$$\Delta = \frac{L}{\frac{1}{2}\rho_\infty U_\infty} \cdot \frac{\mathrm{d}P_s}{\mathrm{d}x} \tag{4}$$

where L denotes the length of the plate, ρ_∞, the density of air in the free stream, and U_∞ is the free-stream velocity. In view of the fact that the pressure varies almost linearly with the distance from the leading edge, the pressure gradient can be assumed constant. Then the value of the quantity Δ is 0·056 in the free stream and 0·106 on the surface of the plate and it is independent of the air speed. It is noted from Fig. 8 that the static pressure also varies slightly across the boundary layer. The difference, however, is so small that it will be considered consistent with the boundary layer approximations to neglect it.

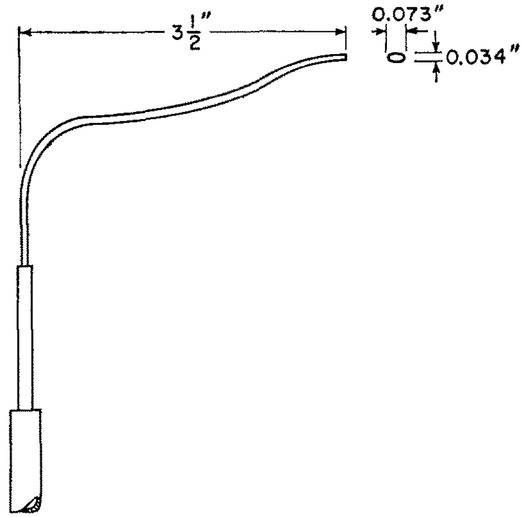


FIG. 9. Boundary layer probe.

The velocity profile of the boundary layer was measured by a total-pressure probe with a rectangular face of 0·034 in × 0·073 in (Fig. 9). The probe was screwed to one end of a long stem which was fixed to a traversing mechanism. The traversing mechanism could be moved by a micrometer screw provided with a graduated head. In such a manner the distance of the probe from the flat plate could be determined to within 0·001 in.

The static pressure at each position in the boundary layer was not measured. In calculating the velocity profile, the static pressure measured at 1 in from the flat plate was used because the static pressure was approximately constant across the boundary layer.

The velocity profile and the heat transfer measurements were made immediately one after the other, in other words, the profiles were taken when the flat plate was hot. One profile was taken for each heat transfer determination. A typical laminar velocity profile is shown in Fig. 10(a), and a typical turbulent one, in Fig. 10(b). In Fig. 10(a), the profile is shown by a plot of u/U_∞ in terms of the Blasius parameter,

$$\eta = y(U_\infty/\nu x)^{1/2}, \tag{5}$$

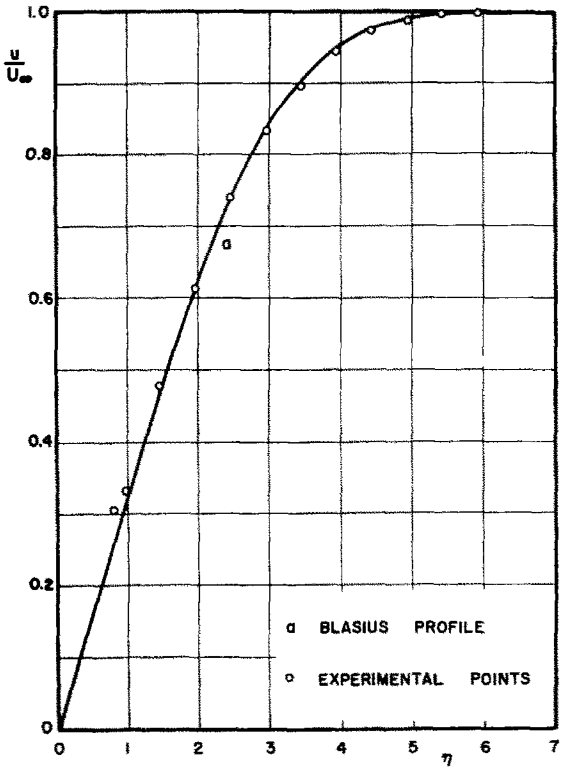


FIG. 10(a) Typical laminar velocity profile (hot) $Re = 123\ 000$ $Tu = 1{\cdot}0$ per cent.

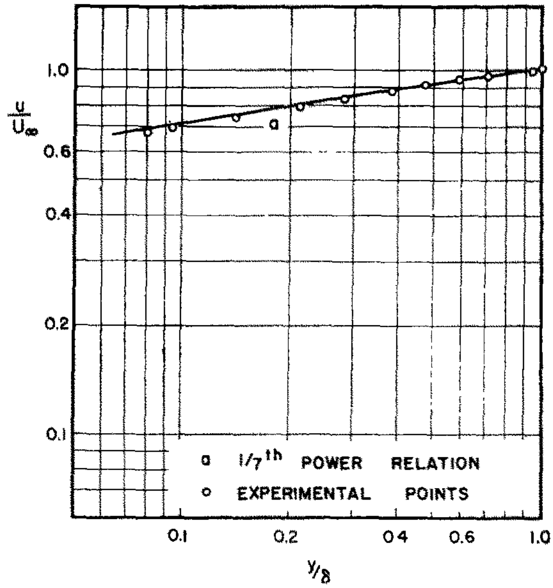


FIG. 10(b) Typical turbulent velocity profile (hot) $Re = 289\ 000$ $Tu = 3{\cdot}7$ per cent.

u being the velocity at distance y from the flat plate with ν denoting the mean value of the kinematic viscosity of air. It is seen that the velocity closely approximates a Blasius profile.

In Fig. 10(b), a turbulent velocity profile is shown by plotting u/U_∞ in terms of y/δ_m in logarithmic co-ordinates; here δ_m is the measured boundary layer thickness taken at the $u/U_\infty =$ 99 per cent position. The shape of the profile agrees very well with the 1/7th power relation. In most runs in the turbulent region, the calculated boundary layer thickness,

$$\delta = 0{\cdot}37\, x(U_\infty x/\nu)^{-0{\cdot}2}, \tag{6}$$

agrees with the measured one.

In order to gain a better idea of the character of the boundary layers over the entire range of Reynolds numbers, six measurements of u/U_∞ are shown in terms of y/δ_m in Fig. 11 for six

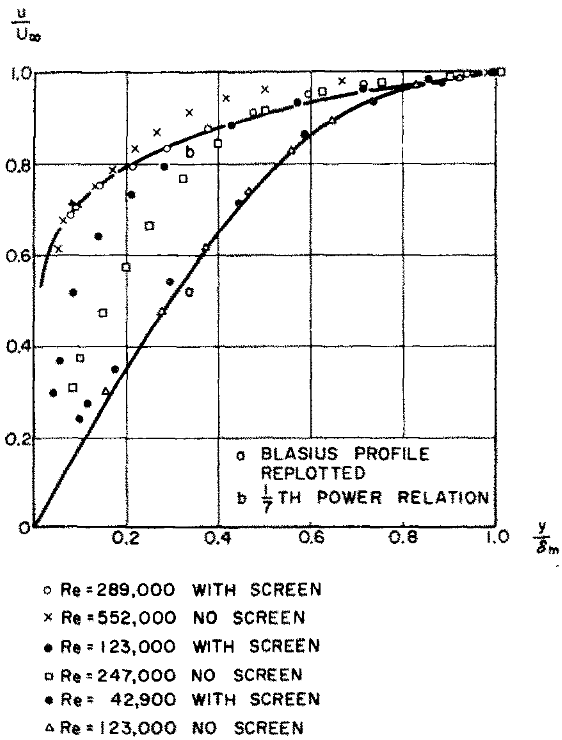


FIG. 11. Velocity profiles in boundary layer.

Reynolds numbers. In Fig. 11, curve a is the Blasius profile replotted, taking

$$\delta = 5{\cdot}0(\nu x/U_\infty)^{1/2} \tag{7}$$

as the boundary layer thickness and curve b is the 1/7th power relation. The character of the three regimes, laminar, transition and turbulent is clearly demonstrated. Edwards and Furber gave a similar illustration except that they used the cubic parabolic approximation for the Blasius profile. It may be remarked here that the cubic parabolic profile would lie slightly below the curve a in Fig. 11.

It is well known that transition from laminar to turbulent flow in the boundary layer can be clearly distinguished by a sudden and large increase in the boundary layer thickness. The dimensionless boundary layer thickness,

$$\delta(U_\infty/\nu x)^{1/2}, \tag{8}$$

is constant for laminar flow and the constant is approximately equal to 5, as can be seen from equation (7). Hansen [19] measured the boundary layer thickness along a flat plate in parallel flow at zero incidence, and found that the sudden change occurs at a Reynolds number,

$$U_\infty x/\nu = 320\,000. \tag{9}$$

His graph is reproduced in Fig. 12 which also contains the present results. It is seen that in the present experiment, the transition occurs at lower Reynolds numbers. This is evidently an influence of the free-stream turbulence.

In all the velocity profile explorations, the distance from the flat plate was always measured at the geometric center of the probe. It was shown experimentally by Young and Mass [20] that a displacement is associated with the geometric center of the face of the probe in measuring the total pressure in a region with transverse total-pressure gradient, such as in the boundary layer. A successful correction factor was reported for circular-faced probes but no reliable information was obtained for rectangular shapes. At any rate, applying such a displacement correction for the probe changes the shape of the profile only slightly, leaving its main character unaffected.

In addition to the displacement correction of the geometric center, MacMillan [21] also experimented on the wall effect when the total pressure at positions very close to a wall was to be taken. He also found a reliable correction for u/U_∞, but the correction was so small in the present case that it was entirely neglected.

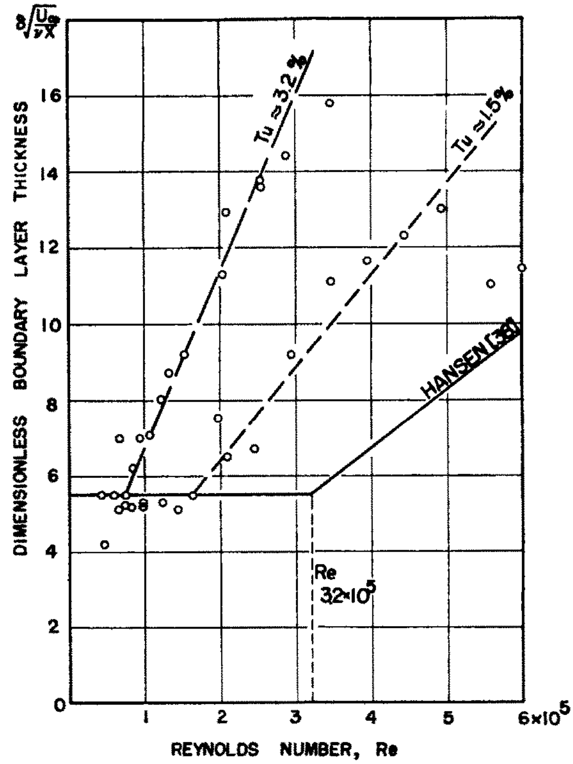


FIG. 12. Transition Reynolds numbers.

It is thus believed that the uncorrected velocity profiles as presented in Figs. 10(a), (b) and 11 provide a fundamentally justifiable basis for the determination of the character of the boundary layers present in each case.

Velocity profiles were also measured for four Reynolds numbers when the flat plate was cold; no difference was found between them and those measured when the plate was hot if the kinematic viscosity of air was taken at the free-stream temperature.

5. EVALUATION OF RESULTS

The free-stream velocity U_∞, in m/s, was calculated by the following formula:

$$U_\infty = C_1(q_\infty/\rho_\infty)^{1/2} \tag{10}$$

where q_∞ denotes the dynamic head in mm H_2O, ρ_∞ denotes the density of air in kg/m³ and the constant C_1 is a conversion factor which has a value of

$$C_1 = 4{\cdot}3897 \text{ kg}^{1/2}/\text{m}^{1/2} \text{ s } (\text{mmH}_2\text{O})^{1/2}. \quad (11)$$

The density ρ_∞, in kg/m³, was calculated from

$$\rho_\infty = C_2 \times (P_{s\infty}/T_{a\infty}), \quad (12)$$

where $P_{s\infty}$ denotes the static pressure in mm Hg, $T_{a\infty}$ denotes the air temperature in °K and the constant C_2 equals to

$$C_2 = 0{\cdot}46447 \text{ (kg °K)/m}^3 \text{ (mmHg)}. \quad (13)$$

The Reynolds and Nusselt numbers were calculated in the following way. It is recalled that in order to justify the assumption of incompressibility, the difference between the surface temperature T_w and the free-stream air temperature $T_{a\infty}$ must be made small compared with the absolute temperature of the flow field, so that

$$(T_w - T_{a\infty})/T_{a\infty} \to 0. \quad (14)$$

When the surface temperature is different from the free-stream air temperature, there must be a variation in the fluid properties appearing in the Nusselt and Reynolds numbers. If the temperature difference is not too great, as in the present case, it is customary to correct for the influence of the variation in the thermodynamic properties with the temperature across the boundary layer by employing the integrated mean values of these properties. Thus Reynolds and Nusselt numbers were calculated from

$$Re = U_\infty x/\nu, \quad (15)$$

$$Nu = \alpha x/k, \quad (16)$$

where x denotes the distance from the leading edge, α denotes the local coefficient of heat transfer and ν and k represent the integrated mean values of the kinematic viscosity and the thermal conductivity of air respectively. The integrated mean values are defined by

$$\nu = \frac{1}{T_w - T_{a\infty}} \int_{T_{a\infty}}^{T_w} \nu_T(T)\mathrm{d}T, \quad (17)$$

$$k = \frac{1}{T_w - T_{a\infty}} \int_{T_{a\infty}}^{T} k_T(T)\mathrm{d}T \quad (18)$$

where $\nu_T(T)$ and $k_T(T)$ are the kinematic viscosity and the thermal conductivity of air at the temperature T, respectively. The numerical values of $\nu_T(T)$ and $k_T(T)$ have been interpolated from the NBS-NACA Tables of Thermodynamic Properties of Gases [22].

The local coefficient of heat transfer α was directly calculated from the electrical measurement of the power input to the heater and the temperature difference. It is true that the local values of α were measured over a length of 0·50 in, i.e. the heater width. However, only a negligible error was expected because the heaters were sufficiently far downstream from the leading edge. In order to justify this point, a simple estimation of errors is provided below.

For the laminar heat transfer, the local coefficient of heat transfer α_1, at the center line of the heater is given by

$$\alpha_1(x) \sim (x)^{-0{\cdot}5}. \quad (19)$$

If $2b$ denotes the width of the heater, the mean value of the coefficient over the heater width is given by

$$\alpha(x, b) \sim \frac{1}{2b} \int_{x-b}^{x+b} (x)^{-0{\cdot}5}\, \mathrm{d}x. \quad (20)$$

The ratio of the local value equation (19), to the mean value, equation (20), can be written as

$$\alpha_1/\alpha = \tfrac{1}{2}\{[1 + (b/x)]^{0{\cdot}5} + [1 - (b/x)^{0{\cdot}5}]\} \approx 1 - (1/8)\,(b/x)^2. \quad (21)$$

because $b/x \ll 1$. For the front heater, $b/x =$ 0·0388, and the ratio becomes

$$\alpha_1/\alpha \approx 0{\cdot}999 \quad (22)$$

and for the back heater the value of the ratio is even closer to unity because b/x is smaller.

For the turbulent heat transfer, the local coefficient of heat transfer α_1, at the center line of the heater is expressed by

$$\alpha_1(x) \sim (x)^{-0{\cdot}2} \quad (23)$$

while the ratio of the local value to the mean value becomes

$$\alpha_1/\alpha \approx 1 - (1/25)\,(b/x)^2. \quad (24)$$

The error is even smaller than that in the laminar case. The preceding calculations thus justify the taking of the direct measurement of the heat transfer as the local value.

If V, I, and A denote the measured voltage, current and the mean area of the heater respectively, then the Nusselt number can be expressed as

$$Nu = \alpha x/k = 1{\cdot}01(VI)\, x/A\Delta Tk \qquad (25)$$

where $\Delta T = T_w - T_{a\infty}$. The mean area is the arithmetic mean value of the area of the slot on the plate surface and that of the exposed surface of the heater. Its value is $A = 35{\cdot}61$ cm^2 for both heaters. The factor 1·01 in equation (25) was deduced from the measuring circuit, as explained in Ref. 1. The accuracy of the determination of the Nusselt and Reynolds numbers was about 1·5 per cent.

The radiation correction was calculated from the following equation:

$$P_r = \epsilon C_b A\ \{(T_w/100)^4 - (T_{a\infty}/100)^4\} \qquad (26)$$

where $C_b = 5{\cdot}77$ W/m$^2C^4$ and the emissivity ϵ was assumed to be 0·08.

6. CORRECTION DUE TO NON-ISOTHERMAL SURFACE

As already mentioned, the plate surface temperature was not uniform near the leading edge, and it also differed slightly from the heater surface temperature near the heaters. For the convenience of calculating the corrections, the plate surface temperature was assumed to have a discontinuity at 1·5 in from the leading edge and a second discontinuity at the front edge of each heater (Fig. 13).

The local coefficient of heat transfer from a flat plate with an unheated starting length was derived by Eckert [16] for a laminar boundary layer and by Rubesin [17] for a turbulent boundary layer. The coefficient $\alpha(L, x)$ is a function of the unheated length L and the distance from the leading edge x. Consequently $\alpha(0, x)$ denotes the coefficient of heat transfer without an unheated length, i.e. for the case when the momentum and thermal boundary layers both begin at the leading edge. The ratio between $\alpha(0, x)$ and $\alpha(L, x)$ denoted by

$$f(L, x) = \alpha(L, x)/\alpha(0, x) \qquad (27)$$

was calculated by Eckert and by Rubesin who found the following two expressions:

$$f(L, x) = \{1 - (L/x)^{0{\cdot}75}\}^{-1/3}{}_{\text{laminar}}, \qquad (28)$$

$$f(L, x) = \{1 - (L/x)^{39/40}\}^{-7/39}{}_{\text{turbulent}}. \qquad (29)$$

The present flat plate did not possess a portion which was entirely unheated; consequently the above two correction factors could not be applied directly to the present case.

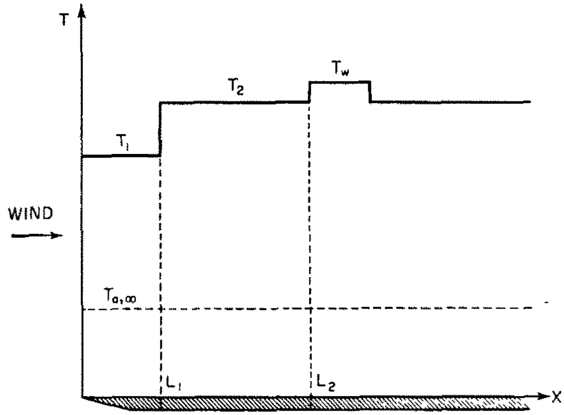


FIG. 13. Idealized temperature distribution on plate.

It was shown by Sogin [23] that the rate of heat transfer from a flat plate with a step-wise discontinuity in the surface temperature can be obtained as the sum of two simple solutions: (1) a simple solution for a heated plate at uniform temperature T_1, see Fig. 13; and (2) a simple solution for a heated step at temperature $T_2 - T_1$ with the step starting at L_1. The superposition of simple solutions can be extended to the case of n steps. In the present case, there are two steps for each heater. Therefore the coefficient of heat transfer $\alpha(L, x)$ can be expressed as

$$\alpha(L, x) = \frac{\alpha(0, x)}{T_w - T_{a\infty}}\ \{(T_1 - T_{a\infty}) + (T_2 - T_1)f(L_1, x) + (T_w - T_2)f(L_2, x)\}. \qquad (30)$$

Since the function f has a singularity at $x = L$, the integrated mean value of the left-hand side of equation (30) over the heater width must be taken in order to obtain the local coefficient of heat transfer. Thus

$$Nu(L, x) = Nu(0, x)\,.\,C. \qquad (31)$$

where

$$C = \frac{1}{T_w - T_{a\infty}} \left\{ (T_1 - T_{a\infty}) + \frac{T_2 - T_1}{2b\alpha(0, x)} \int_{x-b}^{x+b} \alpha(0, x) \,.\, f(L_1, x) \mathrm{d}x + \frac{T_w - T_2}{2b\alpha(0, x)} \int_{x-b}^{x+b} \alpha(0, x) \,.\, f(L_2, x) \mathrm{d}x \right\}. \quad (32)$$

The evaluation of the two integrals will be given in detail in the Appendix.

Together with the radiation correction, the corrected Nusselt number is given by

$$Nu = (1{\cdot}01\ VI - P_r)\ x/A(\Delta T)kC \quad (33)$$

All the Nusselt numbers were calculated from equation (33).

It is noted that the correction factor f was derived only for the laminar and turbulent boundary layers. In the transition region, half of the experimental points were corrected according to the laminar correction, and the other half, according to the turbulent correction.

7. EXPERIMENTAL RESULTS

The experimental results are given in Table 1 and Fig. 14. In Fig. 14 the results are shown as plots of the Nusselt number in terms of the Reynolds number in logarithmic co-ordinates. For the purpose of comparison, the well-known relations due to Pohlhausen [24, 25], Prandtl [25] and von Kármán [25, 26] are shown as lines a, b, and c respectively. Line a corresponds to Pohlhausen's equation

$$Nu = 0{\cdot}295\ Re^{1/2} \quad (34)$$

for laminar flow and $Pr = 0{\cdot}70$. Line b corresponds to Prandtl's semi-empirical relation for turbulent flow and $Pr = 0{\cdot}70$:

$$Nu = 0{\cdot}0236\ Re^{0{\cdot}8}. \quad (35)$$

Finally, line c corresponds to von Kármán's corresponding semi-empirical relation

$$Nu = 0{\cdot}0241\ Re^{0{\cdot}8}. \quad (36)$$

In the laminar region, the experimental points agree with Pohlhausen's solution to within $\pm 2{\cdot}0$ per cent, and the reader might recall that Pohlhausen's solution was derived for a free stream of zero turbulence intensity. In the present experiment, the turbulence intensity varied from 0·75 to 3·82 per cent, as can be seen in Table 1. The agreement between the present experimental results and the theoretical prediction leads to the conclusion that the free-stream turbulence does not affect the local coefficient of heat transfer across a laminar boundary layer on a flat plate up to a turbulence intensity of 3·82 per cent. In other words, the random fluctuations in the free-stream velocity do not cause any measurable effect on the laminar boundary layer for a turbulence intensity in the present range. This constitutes a marked difference from the case of cylinders and spheres.

In the transition region, a great influence was detected as expected.

The effect of the free-stream turbulence on the position of the point of transition was first studied by Dryden [27]. Dryden examined the flow in the boundary layer on a flat plate and obtained initial results on the behavior of the

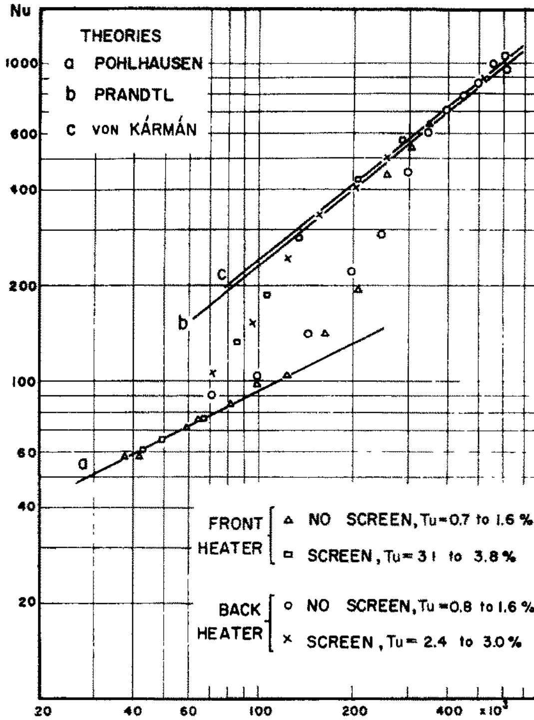


FIG. 14. Local heat transfer from flat plate; negligible pressure gradient.

Table 1. *Experimental results*
(*Negligible pressure gradient*)

Air speed U_∞ (m/s)	Temp. difference ΔT (°C)	Measured power 1·01 (VI) (W)	Radiation correction P_r (W)	Correction for non-isothermal condition C	Corrected Nusselt number Nu	Reynolds number $Re \times 10^{-3}$	Turbulence intensity Tu (%)
First	Heater						
4·00	78·20	2·8696	0·1965	1·025	58·5	37·8	1·60
4·42	79·00	2·8227	0·1974	1·019	57·2	41·9	1·60
4·57	76·46	2·9077	0·1942	1·028	60·4	42·9	3·82
5·25	75·30	3·1339	0·1915	1·022	66·8	49·2	0·90
6·25	78·48	3·4782	0·1970	1·029	71·3	59·1	0·75
6·75	79·21	3·7087	0·1976	1·023	76·1	64·0	0·75
7·06	74·50	3·6624	0·1887	1·063	76·6	66·3	3·17
8·77	73·39	3·9486	0·1884	1·048	85·2	81·9	0·81
10·72	73·19	4·5351	0·1879	1·056	98·0	99·9	1·09
13·18	73·55	5·1188	0·1887	1·060	110·2	123·0	1·00
9·03	77·16	6·3200	0·1932	1·043	133·4	85·3	3·26
11·09	76·86	8·5440	0·1923	1·013	187·8	105·0	3·58
17·48	73·60	6·5804	0·1886	1·077	140·6	163·0	1·46
22·19	73·66	9·0068	0·1877	1·073	194·5	208·0	1·38
26·98	70·69	18·9058	0·1820	1·030	446·5	251·0	1·04
32·54	70·28	22·7319	0·1800	1·030	540·8	302·0	1·07
14·17	77·12	12·9424	0·1928	1·018	284·5	134·0	3·37
22·12	74·67	19·6379	0·1881	1·021	445·3	208·0	3·72
31·00	71·42	24·7268	0·1828	1·029	580·7	289·0	3·68
37·89	63·29	23·8750	0·1656	1·039	644·4	347·0	1·15
Second	Heater						
3·94	78·47	2·3262	0·1966	1·011	90·3	71·4	1·37
3·91	79·38	2·7088	0·1983	1·011	105·2	71·1	2·44
5·24	78·12	3·8386	0·1958	1·016	154·2	95·1	2·54
5·42	77·43	2·5637	0·1956	1·013	101·4	97·9	0·89
6·82	76·21	5·7401	0·1925	1·005	242·9	123·0	2·65
8·00	76·89	3·6085	0·1938	1·049	142·0	144·0	0·80
10·89	74·71	5·4310	0·1899	1·049	223·7	196·0	1·10
13·70	75·80	7·1263	0·1917	1·056	290·3	247·0	1·26
16·44	74·41	10·3499	0·1887	1·011	451·8	296·0	1·35
19·44	73·13	13·5958	0·1864	1·012	604·9	348·0	1·50
8·50	75·0	7·7685	0·1900	1·005	336·5	153·0	2·90
11·33	72·75	9·0182	0·1851	1·005	403·2	203·0	2·90
14·33	70·94	11·1454	0·1818	1·014	509·4	256·0	2·92
22·16	72·49	15·8986	0·1853	1·013	713·9	396·0	1·50
24·79	72·60	17·6986	0·1841	1·015	793·1	445·0	1·41
27·65	71·59	19·2864	0·1822	1·014	876·9	495·0	1·25
29·37	68·55	19·1867	0·1764	1·014	907·5	522·0	2·95
30·86	71·39	22·0417	0·1817	1·012	1008·1	522·0	1·07
33·69	68·86	22·2047	0·1767	1·014	1047·1	600·0	0·96
34·24	70·08	20·3622	0·1799	1·015	943·5	610·0	0·95

transition point. These results and others have been summarized by Gazley [28] who correlated the transition Reynolds number with the free-stream intensity. His correlation curves are reproduced in Fig. 15. When compared with Gazley's correlation, it is found that in the present experiment the transition Reynolds numbers were slightly lower for the respective turbulence intensities.

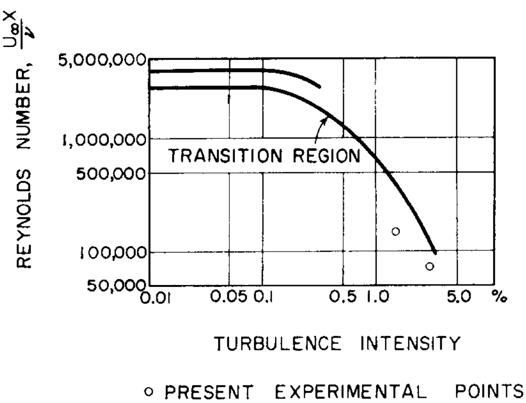


FIG. 15. Effect of free-stream turbulence on boundary layer transition from Gazley [28].

In the turbulent region, the experimental points are more scattered. It can be seen from Fig. 14 that most of the points lie on or in between lines b and c which are Prandtl's and von Kármán's equations respectively.

The largest deviation which appears at the highest Reynolds numbers is about +3·5 per cent from line c and −3·0 per cent from line b. In spite of these deviations, the general trend of the experimental points shows that the local coefficient of heat transfer was not affected by the change of the free-stream turbulence intensity in the range from 0·95 to 3·72 per cent. There is no evidence of any systematic effect of turbulence intensity.

The results of our experiments fully agree with the conclusions reached by Edwards and Furber [10] and Reynolds *et al.* [13]. The discordant results obtained by Sugawara and Sato [12] require further discussion.

As mentioned in section 1, Sugawara and Sato also measured the local coefficient of heat transfer from a flat plate to a turbulent air stream. Their results are replotted in Fig. 16 in which the Nusselt number Nu, instead of the ratio of the Nusselt number to the Reynolds

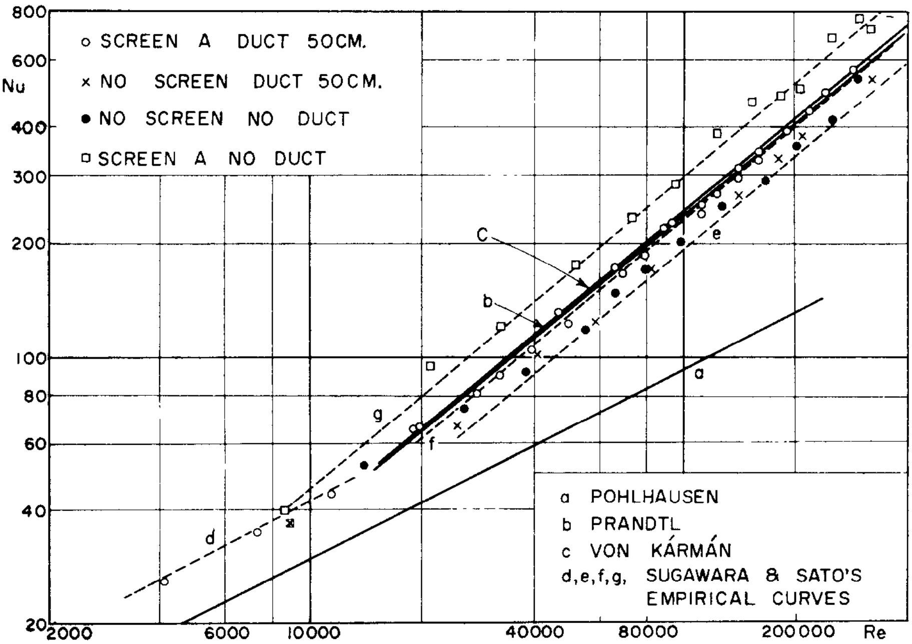


FIG. 16. Experimental results obtained by Sugawara and Sato [12].

number Nu/Re as in the original plot, is drawn in terms of the Reynolds number Re. The dotted lines in Fig. 16 are Sugawara and Sato's empirical expressions of their results for the laminar heat transfer (d) and the turbulent heat transfer (e, f, and g). The solutions due to Pohlhausen for the laminar heat transfer and that due to von Kármán for the turbulent heat transfer are also included. It is quite clear from Fig. 16 that Sugawara and Sato's experimental results in the laminar range deviate from Pohlhausen's solution by a large amount. In the turbulent range, some of their points lie much below von Kármán's equation and some lie considerably above it. However, the slope of line d is the same as that of line a and the slope of lines e, f, and g is the same as that of line c.

Clearly, the general features of their results do not agree with those of the others [10, 11, 13] and of the present investigation, Fig. 14. This divergence might be the result of the questionable accuracy of the non-steady method, or of some defects in their arrangement.

All these measurements, except those due to Sugawara and Sato, lead to the following conclusion: The free-stream turbulence does not exert a local effect on the coefficient of heat transfer through a laminar or a fully developed turbulent boundary layer *on a flat plate*. The only effect due to turbulence is to advance the position of the point of transition.

8. EFFECT OF PRESSURE GRADIENT

As already explained earlier in section 2, there are reasons to suppose that the imposition of a pressure gradient on the flat plate would restore the effect of turbulence intensity on the local rate of heat transfer.

The experimental arrangement employed for testing this supposition was identical with that used previously, except that a plate W was provided, as shown in broken lines in Fig. 3. This produced a large favorable pressure gradient along the test-plate A. The control louvers were removed, and the heat transfer determinations were made at the back heater only.

The static-pressure distribution in the free stream as well as along the plate is shown in Fig. 17 for one air speed. The average pressure

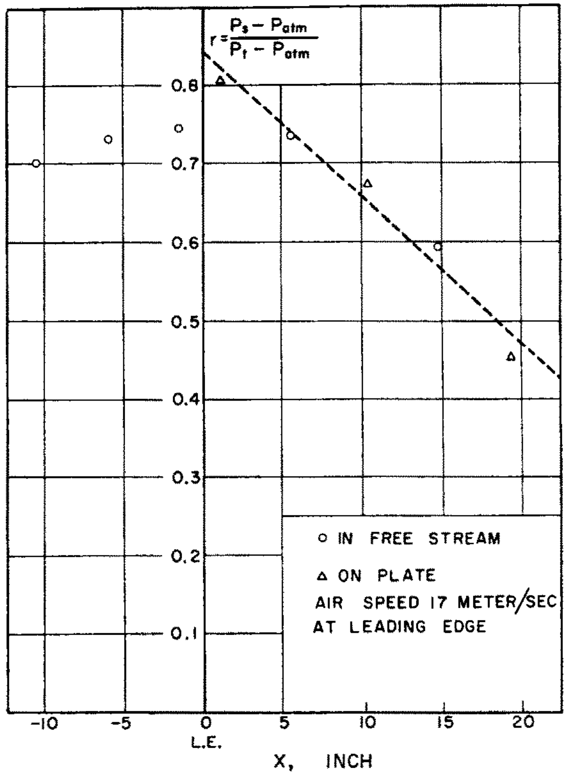


FIG. 17. Variation of static pressure; case of large pressure gradient.

gradient is indicated by the slope of the broken line. The dimensionless average gradient Δ, defined in equation (4) and based on the free-stream velocity at the leading edge has a value $\Delta = 1{\cdot}49$ which represents almost a fifteen-fold increase as compared with the case of a negligible pressure gradient.

The velocity profiles are shown in Fig. 18, in which the velocity profile u/U_∞ is plotted in terms of the dimensionless boundary layer thickness y/δ_m. Curve a is the convergent-channel solution given by Hartree [29] and replotted, taking $\eta = 3{\cdot}25$ as the boundary layer thickness. Curve b is the 1/7th power relation. Not all experimental points are included.

The heat transfer results are given in Table 1 and Fig. 19. It can be seen that in the laminar range, an increase in the turbulence intensity from 0·36 to 1·71 per cent in the range of Reynolds numbers $50\,000 < Re < 100\,000$ causes the Nusselt number to increase uniformly

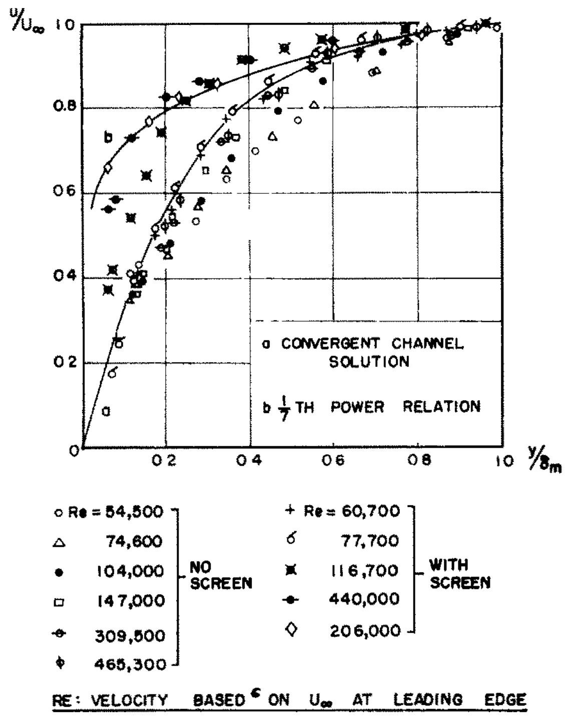


FIG. 18. Velocity profiles in boundary layer; large pressure gradient.

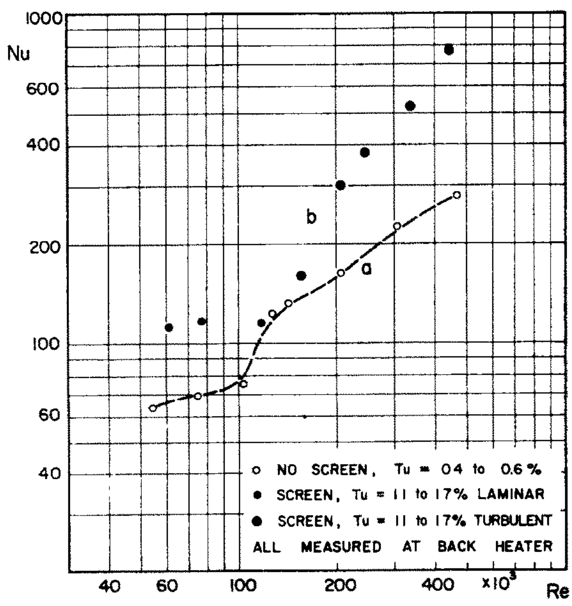


FIG. 19. Local rate of heat transfer with favorable pressure gradient.

by about 65 per cent which is very large and no longer surprising. The abrupt change in the Nusselt number near the Reynolds number $Re = 100\,000$ in Fig. 19 does not correspond to transition, because all the velocity profiles associated with the measurements along the lower line a in Fig. 19 turned out to be laminar in character. In contrast with that, along the upper line b, the velocity profiles below $Re \approx 100\,000$ were laminar, whereas those above $Re \approx 300\,000$ were turbulent, as can be seen from Fig. 18. Thus transition occurred at the higher turbulence intensity, but for the lower turbulence intensity, line a, the addition of the favorable pressure gradient seems to have stabilized the flow in the boundary layer and prevented transition in the range of Reynolds numbers covered by the experiments. Both the Nusselt and Reynolds numbers were based on the distance from the leading edge. The free-stream velocity at the leading edge was used to calculate the Reynolds number.

ACKNOWLEDGEMENTS

The authors wish gratefully to acknowledge the financial assistance received from the then NACA (now NASA) under a contract with Brown University. They also wish to record their thanks to Mr. C. C. Cometta, then Research Assistant at Brown University, for his help with the design of the flat plate and with the conduct of the experimental work. Mr. D. I. Paddison, former Research Assistant at Brown University, performed all the turbulence measurements.

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APPENDIX

Evaluation of the correction factor for non-isothermal conditions, equation (32)

For the first integral in equation (32), we can take the value calculated at the center line of each heater because the temperature jump occurs at L_1 which is far upstream from the heaters. Therefore the first integral becomes

$$I_1 \approx f(L_1, x).$$

The function $f(L_1, x)$ was given in equations (28) and (29). Thus we obtain the following values:

	Laminar	Turbulent
Front heater	$I_1 = 1{\cdot}14$	$I_1 = 1{\cdot}045$
Back heater	$I_1 = 1{\cdot}08$	$I_1 = 1{\cdot}022$

The second integral cannot be evaluated in the same way since the temperature jump occurs at L_2 which is the front edge of each heater. In the laminar region, the integral becomes

$$I_2 = \frac{1}{2ba(0, x)} \int_{x_1}^{x_2} \gamma x^{-0{\cdot}25} (x^{0{\cdot}75} - x_1^{0{\cdot}75})^{-1/3} \, dx$$

where $x_1 = x - b = L_2$, $x_2 = x + b$ and γ is a constant. Note that the symbol x in the lower and upper limits denotes the distance from the leading edge to the center line of each heater; thus it is a constant. Substituting $z = x^{3/4}$, the integral I_2 becomes

$$I_2 = (x_2^{0\cdot75} - x_1^{0\cdot75})^{2/3}\,(x_2^{0\cdot5} - x_1^{0\cdot5})^{-1}.$$

The numerical values of the integral I_2 are:

for the front heater: $I_2 = 4{\cdot}11$
for the back heater: $I_2 = 4{\cdot}96$.

In the turbulent region, the integral I_2 was evaluated numerically and the values are:

for the front heater: $I_2 = 1{\cdot}41$
for the back heater: $I_2 = 1{\cdot}54$.